Pre-Calculus CP 1 – Section 3.5 Day 2 HW **Name:** Key

Application Word Problems!

1. A deposit of $10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 5 years.

$A = Pe^{rt}$

$A=2 \quad P=1 \quad t=5$

a) What is the annual interest rate for this account?

$2 = e^{r(5)}$

$\ln 2 = 5r$

$r = \frac{\ln 2}{5} \approx .138629 \Rightarrow 13.86\%$

b) Find the balance after 1 year.

$A = 10{,}000\, e^{.138629(1)} \approx \$11{,}486.98$

2. The half-life of radioactive uranium II is 250,000 years. What percent of a present amount of radioactive uranium II will remain after 5000 years?

$A(t) = A_0 e^{kt}$

$\frac{1}{2} = e^{250{,}000 k}$

$k = \frac{\ln \frac{1}{2}}{250{,}000} = -0.0000027725887$

$A = e^{k(5000)} \approx .98623 \Rightarrow 98.62\%$

3. The population of South Carolina (in thousands) from 1990 through 2003 can be modeled by $P(t) = 3499e^{0.0135t}$, where t is the time in years, with t = 0 corresponding to 1990. According to this model, when will the population reach 4.5 million? → 4500 (in thousands)

$4500 = 3499e^{0.0135t}$

$\frac{4500}{3499} = e^{0.0135t}$

$t = \frac{\ln\left(\frac{4500}{3499}\right)}{0.0135} \approx 18.64 \Rightarrow 2008$

$1990+18$

Application Word Problems!

4. In a typing class, the average number N of words per minute typed after t weeks of lessons was found to be

$$N=\frac{157}{1+5.4e^{-0.12t}}$$

a) What is the carrying capacity for this problem?

157 words

b) Find the time necessary to type 50 words per minute

$$50=\frac{157}{1+5.4e^{-0.12t}}$$

$$5.4e^{-0.12t}=\frac{157}{50}-\frac{50}{50}=\frac{107}{50}$$

$$e^{-0.12t}=\frac{107}{50}\cdot\frac{1}{5.4}=.3693$$

$$t=\frac{\ln(.3693)}{-0.12}\approx 7.71 \text{ weeks}$$

5. The relationship between the number of decibels B and the intensity of a sound I in watts per square centimeter is $B=10\log\left(\frac{I}{10^{-16}}\right)$. Determine the intensity of a sound in watts per square centimeter if the decibel level is 125.

$$125=10\log\left(\frac{I}{10^{-16}}\right)$$

$$12.5=\log\left(\frac{I}{10^{-16}}\right)$$

$$10^{12.5}=\frac{I}{10^{-16}}$$

$$I=\frac{10^{12.5}}{10^{16}}=10^{-3.5}\ \frac{w}{cm^2}$$

Application Word Problems!

6. On a day a person is born, a deposit of $50,000 is made in a trust fund that pays 8.75% interest, compounded continuously.

$A = Pe^{rt}$ .0875

a) Find the balance on the person's 35th birthday.

$50{,}000 = e^{.0875(35)}$ ≈ $1,069,047.14

b) How much longer would the person have to wait for the balance in the trust fund to double over the amount they have when they're 35?

$2{,}138{,}094.28 = 50{,}000e^{.0875t}$

$42.76 = e^{.0875t}$

$t = \frac{\ln(42.76)}{.0875} \approx 42.92 \Rightarrow 43$

$43 - 35 =$ 8 more years

7. Let Q represent a mass of plutonium 241 in grams, whose half-life is 14.4 years. The quantity of plutonium 241 present after t years is given by $Q = 100\left(\frac{1}{2}\right)^{t/14.4}$

a) Determine the initial quantity

$Q = 100\left(\frac{1}{2}\right)^0 =$ 100 grms

b) Determine the quantity present after 10 years.

$100\left(\frac{1}{2}\right)^{10/14.4}$ ≈ 61.795 grms

Application Word Problems!

8. The antler spread **a** (in inches) and shoulder height **h** (in inches) of an adult male American elk are related by the model $h = 116\log(a+40) - 176$. Approximate the shoulder height of a male American elk with an antler spread of 55 inches.

$$h = 116\log(55+40) - 176$$
$$= 116\log(95) - 176 \approx 53.42 \text{ in}$$

9. Mrs. Collins is making cookies and her kids love to eat them right away! She bakes them in a u_0 350 degree (F) oven and when they are done she cools them in a T 70 degree (F) room. Brady comes over and tries to take a bite one minute after she took them out of the oven but she shoos his hand away! They are much too hot to eat at a roasty 270 degrees (F)! He asks, "When can I come back and eat them?" If Mrs. Collins considers 150 degrees to be acceptable, what time should she tell him to return?

$$u(t) = T + (u_0 - T)e^{kt}$$
$$270 = 70 + (350-70)e^{k(1)}$$
$$\frac{200}{280} = \frac{5}{7} = e^{k}$$
$$k = \ln\frac{5}{7}$$

$$150 = 70 + (280)e^{\left(\ln\frac{5}{7}\right)t}$$
$$\frac{80}{280} = \frac{2}{7} = e^{\left(\ln\frac{5}{7}\right)t}$$
$$\ln\frac{2}{7} = \left(\ln\frac{5}{7}\right)t$$
$$t = \frac{\ln\frac{2}{7}}{\ln\frac{5}{7}} \approx 3.7 \Rightarrow 4 \text{ mins}$$

10. Suppose a body is u_0 83°F at 10 PM and that the air temperature around it is T 42°F. After one t hour the body is found to be 76°F. Assuming the Estimate the time of death.

$$76 = 42 + (83-42)e^{k(1)}$$
$$34 = 41e^{k}$$
$$k = \ln\left(\frac{34}{41}\right)$$

Body temp?

$$83 = 42 + (98.6 - 42)e^{\ln\left(\frac{34}{41}\right)t}$$
$$41 = 56.6e^{\ln\left(\frac{34}{41}\right)t}$$
$$t = \frac{\ln\left(\frac{41}{56.6}\right)}{\ln\left(\frac{34}{41}\right)} = 1.72 \text{ hrs}$$

$\therefore$ 8:17 pm

Application Word Problems!

9. Detectives respond to a call at Dunkin Donuts made by Peter at exactly 5:10 AM. When they arrive Peter is panicked and visually upset. He says she arrived at work around 5 AM to open the store with a fellow worker. Peter has a witness in her father who says he dropped him off at exactly 5 AM. Peter tells the police that when he entered the store his fellow co-worker was already dead. The coroner arrives to take some temperatures and finds that the body is 85°F u_0 and the room its in is kept at a constant 68°F. These temperatures are taken at exactly 5:55 AM. Two hours later the coroner takes the second temperature reading. He finds the body to be 74°F and the room to still be 68°F. Should the police consider Peter a suspect?

$u(t)$ T

$$u(t) = T + (u_0 - T)e^{kt}$$

$$74 = 68 + (85-68)e^{k(2)}$$

$$6 = 17e^{2k}$$

$$k = \frac{\ln\left(\frac{6}{17}\right)}{2}$$

$$85 = 68 + (98.6-68)e^{\left(\frac{\ln\left(\frac{6}{17}\right)}{2}\right)t}$$

$$\frac{17}{30.6} = e^{\left(\frac{\ln\left(\frac{6}{17}\right)}{2}\right)t}$$

$$t = \frac{\ln\left(\frac{17}{30.6}\right)}{\left(\frac{\ln\left(\frac{6}{17}\right)}{2}\right)} = 1.13 \text{ hrs}$$

Peter is not a suspect!